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Hanbury-Brown and Twiss Intensity Correlations of Parabosons

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Abstract

This paper shows that in intensity correlation measurements there will be clear and unambiguous signals that new-physics particles are, or aren't, parabosons. For a parabosonic field in a dominant single-mode, there is a diagonal P_p-representation in the $|\alpha_{even,odd}>$ coherent state basis. It is used to analyze zero-time-interval intensity correlations of parabosons in a maximum-entropic state. As the mean number of parabosons decreases, there is a monotonic reduction to $\frac{2}{p}$ of the constant bosonic "factor of two" proportionality of the second-order versus the squared first-order intensity correlation function.

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Statistics based correlations, in particular second and higher-order intensity correlations [1,2], have proven to be very useful in interferometry measurements and in dynamical investigations of new phenomena in areas ranging from astrophysics to nuclear and high energy physics [3]. It has been shown that relativistic, local quantum field theory allows particles which obey parastatistics, an elegant generalization of bose and fermi statistics based on the permutation group [4]. While a use for parastatistics in the description of natural phenomena has yet to be found, a wealth of new but poorly studied discoveries have been recently made in astrophysics/cosmology including an inflation era, dark matter, and dark energy. New particles are expected to be produced in experiments at the Large Hadron Collider such as Higgs particles associated with electroweak symmetry breaking (EWSB) and supersymmetry particles associated with the energy-scale inequality $\Lambda_{EWSB} << \Lambda_{Planck}$. Therefore, for incisive analyses, it is both timely and important to know both the selection rules and the novel statistical effects/correlations [4,5] for the purpose of detecting the presence of effective or underlying fundamental quanta obeying parastatistics. This paper shows that by intensity correlation measurements there will be definitive signals that new-physics particles are, or aren't, parabosons.

For single-mode parabosons (pB's), we introduce an index $i = even, odd$. The single-mode pB commutation relations [4,6] are $[a, \{a^\dagger, a\}] = 2a$ and its adjoint, with $\widehat{N}_B \equiv \widehat{H}_B - \frac{p}{2} \equiv \frac{1}{2}\{a^\dagger, a\} - \frac{p}{2}$. In the occupation number basis, the states are

$$|n_i >= \frac{1}{\sqrt{(n_i)_p!}}(a^\dagger)^{n_i}|0 > \tag{1}$$

with $n_e = 2N, n_o = 2N+1; N = 0, 1, \ldots$; and where the p-factorials are defined

$$
\begin{aligned}
(n_e)_p! = (2N)_p! &\equiv 2N(2N-2+p)(2N-2)\cdots 4(2+p)2p\,;\ (0)_p! \equiv 1 \\
&= (2N)!!(2N-2+p)!! \qquad (2)\\
&= \{2^N N!\}\{2^N \tfrac{\Gamma(N+\frac{p}{2})}{\Gamma(\frac{p}{2})}\}
\end{aligned}
$$

$$
\begin{aligned}
(n_o)_p! = (2N+1)_p! &\equiv (2N+p)2N(2N-2+p)\cdots 4(2+p)2p \\
&= (2N)!!(2N+p)!! \qquad (3)\\
&= \{2^N N!\}\{2^{N+1} \tfrac{\Gamma(N+1+\frac{p}{2})}{\Gamma(\frac{p}{2})}\}
\end{aligned}
$$

Note $(0)_p! = 1$, $(1)_p! = p$, and $(n_o)_p! = (2N+1)_p! = (2N+p)(2N)_p!$. Mathematically, in counting the n_p integers, to reach the next odd (even) n_p integer, one adds $p(1)$ to the usual integer. For fixed-order $p = 1, 2, \ldots$ parabosons, this $(n)_p!$ product of n factors is analogous to the Gibbs-factorial $n!$ for ordinary bosons ($p = 1$). For creation of one more pB, the next to vacuum position has $p(1)$ openings when n_{even} (n_{odd}) parabosons are already present; the other positions have 1 opening.[2] To make an odd number of $a^\dagger$ insertions depends on p but to make an even number does not. Because $a^\dagger, a$ are raising and lowering operators, all pB single-mode $a^\dagger, a$ commutation relations in this $|n_{e,o}>$ basis follow from (1).

For single-mode parafermions (pF's), in the occupation number basis, there is a band of "p+1" states. The pF commutation relations are $[c,[c^\dagger, c]] = 2c$ and its adjoint, with $\widehat{N}_F = \widehat{H}_F + \frac{p}{2} \equiv \frac{1}{2}[c^\dagger, c] + \frac{p}{2}$. The pF number states are equivalently

$$
|n>_F = \frac{1}{\sqrt{\{n\}_p!}}(c^\dagger)^n|0>_F = \frac{1}{\sqrt{\{p-n\}_p!}}(c)^{p-n}|p>_F;\ 0 \leq n \leq p \qquad (4)
$$

[2]This characterization follows from $[a,(a^\dagger)^{2n}] = 2n(a^\dagger)^{2n-1}$ and $[a,(a^\dagger)^{2n+1}] = (a^\dagger)^{2n}(2n + [a, a^\dagger])$ since $aa^\dagger|0>= p|0>$.

with $c|0>_F = c^\dagger|p>_F = 0$. Analogous to the $(n)_p!$ for pB's, for pF's

$$\begin{aligned} \{n\}_p! &\equiv \{n(p-n+1)\}\{(n-1)(p-n+2)\}\cdots\{2(p-1)\}\{1\,p\} \\ &= n!\tfrac{p!}{(p-n)!} \end{aligned} \tag{5}$$

which is a product of n bi-factors. Note $\{0\}_p! = 1$ and $\{1\}_p! = p$. In constructing a specific $|n>_F$ state from the adjacent $|n-1>_F$ state, the additional $\{n(p-n+1)\}$ bi-factor is the product of the number of ways to insert the extra " $c^\dagger$ " in the first expression in (4), times the number of ways to remove a " c " in the second expression. The Hamiltonian defines the direction $|0>_F$ to $|p>_F$.

There are important consequences of $(n)_p!$ and $\{n\}_p!$: For a number "n_I" of parabosons initially in the mode, we let $Prob_B(\frac{a}{n_I} \rightarrow \frac{b}{n_F})$ be the probability that in some fixed time interval a system is found to make a transition[3] from a state "a" to a state "b" by the emission or absorption of a single pB $\check{\gamma}$. For initially $n_{even} = 2N$ pB's, the decay probability is

$$Prob_B(\frac{a}{2N} \rightarrow \frac{b}{2N+1}) = (2N+p)\ A \tag{6}$$

whereas for excitation from the state "b" to "a", in the same time interval, the absorption probability is

$$Prob_B(\frac{b}{2N} \rightarrow \frac{a}{2N-1}) = (2N)\ A \tag{7}$$

By time-reversal invariance, the constant A is the same for emission and absorption. Hence, versus ordinary bosons, for the system initially in an n_{even}-mode, there is a p-dependent enhanced $\check{\gamma}$ stimulated-emission. Likewise, for initially $n_{odd} = 2N+1$ pB's, for emission/absorption

$$Prob_B(\frac{a}{2N+1} \rightarrow \frac{b}{2N+2}) = (2N+2)\ A;\ \ Prob_B(\frac{b}{2N+1} \rightarrow \frac{a}{2N}) = (2N+p)\ A \tag{8}$$

[3]A "soft accent" denotes a single para-particle.

Hence, for ordinary bosons, $p=1$, there is stimulated emission. However, for the system initially in an n_{odd}-mode, for $p=2$ the probabilities are equal; but for $p>2$ pB's there is a p-dependent enhanced $\check{\gamma}$ absorption versus emission. In summary, from consideration of single-mode pB statistics versus bose statistics, for a system in an initial pB n_{even}-mode (n_{odd}-mode), there is a p-dependent enhanced $\check{\gamma}$ stimulated-emission by the system (stimulated-absorption by the system); otherwise there is no p-dependence in these transition probabilities. This is different from the generic bunching signatures of ordinary bosons!

Also, in contrast, for parafermions the statistics-favored transition probabilities are towards the "half-full/half-empty" $|n_{mid}>$ parafermion-state(s) at the middle of the pF band of "$p+1$" states, due to the bi-factors and time-reversal invariance. From $\{n\}_p!$, for p_{even} there is a single $|n_{mid}>=|\frac{p_{even}}{2}>$ state but for p_{odd} there is a pair of mid-band states $|n_{mid,high/low}=\frac{p_{odd}\pm 1}{2}>$. For ordinary fermions, there are only these two mid-band states:

For a number "n_I" of pF's initially in the mode, we let $Prob_F(\frac{a}{n_I}\rightarrow\frac{b}{n_F})$ be the probability that in some fixed time interval a system is found to make a transition from a state "a" to a state "b" by the emission or absorption of a single pF $\check{\nu}$. If initially $n_I>0$ is in the less-than-half-filled part of the pF band with $n_I<(\frac{p_{even}}{2},\frac{p_{odd}-1}{2})$, then the ratio of probabilities for emission of a single pF $\check{\nu}$ by the system versus $\check{\nu}$ absorption is greater than one,

$$\frac{Prob_F(\frac{a}{n_I}\rightarrow\frac{b}{n_I+1})}{Prob_F(\frac{b}{n_I}\rightarrow\frac{a}{n_I-1})}=\frac{(n_I+1)(p-n_I)}{n_I(p-n_I+1)}>1;\ 0<n_I<(\frac{p_{even}}{2},\frac{p_{odd}-1}{2}) \tag{9}$$

On the other hand, if initially $n_I<p$ is in the more-than-half-filled part of the pF band with $n_I>(\frac{p_{even}}{2},\frac{p_{odd}+1}{2})$, then the ratio of probabilities for emission of a single pF $\check{\nu}$ by the system

versus $\check{\nu}$ absorption is less than one,

$$\frac{Prob_F(\frac{a}{n_I} \rightarrow \frac{b}{n_I+1})}{Prob_F(\frac{b}{n_I} \rightarrow \frac{a}{n_I-1})} = \frac{(n_I+1)(p-n_I)}{n_I(p-n_I+1)} < 1;\ p > n_I > (\frac{p_{even}}{2}, \frac{p_{odd}+1}{2}) \tag{10}$$

As the pF-mode approaches the the middle of the pF band, this is monotonically a smaller statistics effect. Simultaneously, if one ignores the differences in the A factors, the $\check{\nu}$ transition rates themselves increase as the pF-mode approaches mid-band since the mid-band versus end-of-band ratios

$$\begin{aligned} &\frac{Prob_F(\frac{a}{n_{mid}} \rightarrow \frac{b}{n_{mid}\pm 1})}{Prob_F(\frac{a}{n_{end}} \rightarrow \frac{b}{n_{neighbor}})}\Bigg|_{statistics\ factor\ only} = \tfrac{1}{4}(p_{even}+2) > 1;\ p_{even} > 2 \\ &\frac{Prob_F(\frac{a}{n_{mid,low}} \leftrightarrow \frac{b}{n_{mid,high}})}{Prob_F(\frac{a}{n_{end}} \rightarrow \frac{b}{n_{neighbor}})}\Bigg|_{statistics\ factor\ only} = \tfrac{1}{4}(p_{odd}+2+\tfrac{1}{p_{odd}}) > 1;\ p_{odd} > 1 \end{aligned} \tag{11}$$

The pB coherent states [6] can be written in terms of a p-exponential function

$$\begin{aligned} e_p(x) &\equiv \sum_{n=0}^{\infty} \frac{x^n}{(n)_p!} \\ &= e_e(x) + e_o(x);\ e_{e,o}(-x) = \pm e_{e,o}(x). \end{aligned} \tag{12}$$

In terms of the modified Bessel function $I_\nu(x)$, the even and odd terms are

$$e_e(x) = (\frac{x}{2})^{\frac{2-p}{2}} \Gamma(\frac{p}{2}) I_{\frac{p-2}{2}}(x); e_o(x) = (\frac{x}{2})^{\frac{2-p}{2}} \Gamma(\frac{p}{2}) I_{\frac{p}{2}}(x) \tag{13}$$

For $p=1$, $e_{e,o}(x) \rightarrow \cosh x, \sinh x$.

Thereby, with α complex-valued, $a|\alpha >= \alpha|\alpha >$ for

$$\begin{aligned} |\alpha >&= \frac{1}{\sqrt{e_p(|\alpha|^2)}} e_p(\alpha\, a^\dagger)|0> \\ &= \sqrt{P_e(|\alpha|^2)}\,|\alpha_e> + \sqrt{P_o(|\alpha|^2)}\,|\alpha_o> \end{aligned} \tag{14}$$

with

$$\begin{aligned} |\alpha_e >&= \frac{1}{\sqrt{e_e(|\alpha|^2)}} \sum_{N=0}^{\infty} \frac{|\alpha|^{2N}}{(2N)_p!}|2N> \\ |\alpha_o >&= \frac{1}{\sqrt{e_o(|\alpha|^2)}} \sum_{N=0}^{\infty} \frac{|\alpha|^{2N+1}}{(2N+1)_p!}|2N+1> \\ <\alpha_e|\alpha_e &>=<\alpha_o|\alpha_o>= 1; <\alpha_e|\alpha_o>= 0 \end{aligned} \tag{15}$$

In (14), the important $n_{e,o}$ coherent-state-mode probabilities are

$$P_{e,o}(|\alpha|^2) \equiv \frac{e_{e,o}(|\alpha|^2)}{e_p(|\alpha|^2)} \tag{16}$$

These range monotonically in $|\alpha|^2$ from $P_{e,o} \to 1 - \frac{|\alpha|^2}{p}, \frac{|\alpha|^2}{p}$ respectively as $|\alpha|^2 \to 0$, to $P_{e,o} \to \frac{1}{2}$ for $|\alpha|^2 >> 1, \frac{p}{2}$.

Although $a^2|\alpha_{e,o}>= \alpha^2|\alpha_{e,o}>$, it is with a different normalization

$$|\alpha_+ >\equiv \sqrt{2P_e(|\alpha|^2)}\,|\alpha_e >; \ |\alpha_- >\equiv \sqrt{2P_o(|\alpha|^2)}\,|\alpha_o > \tag{17}$$

that $a|\alpha_\pm >= \alpha|\alpha_\mp >$. Note that $|\alpha_\pm >= \frac{1}{\sqrt{2}}[|\alpha > \pm | -\alpha >]$,
$|-\alpha >= \sqrt{P_e(|\alpha|^2)}\,|\alpha_e > - \sqrt{P_o(|\alpha|^2)}\,|\alpha_o >$, and $|\alpha_{e,o} >= \frac{1}{\sqrt{2P_{e,o}(|\alpha|^2)}}[|\alpha > \pm | -\alpha >]$.

There is an associated p-Poisson distribution function, $x = |\alpha|^2$,

$$\mathcal{P}_p(n_i, x) \equiv \frac{x^{n_i}}{(n_i)_p!}\frac{1}{e_p(x)} = | < n_i|\,\alpha > |^2; \ i = even, odd \tag{18}$$

for the probability to be in the nth number-state if the system is in the coherent state $|\alpha >$.

$$\begin{aligned} \mathcal{P}_p(n_i, x) &\to \tfrac{|\alpha|^{2n_i}}{(n_i)_p!}\{1 - \tfrac{|\alpha|^2}{p} + \mathcal{O}(|\alpha|^4)\}, \quad |\alpha|^2 \to 0 \\ &\to \tfrac{2^{\frac{p-1}{2}}\sqrt{\pi}}{\Gamma(\frac{p}{2})}[\tfrac{|\alpha|^{2(n_i - [\frac{p-1}{2}])}}{(n_i)_p!}\exp(-|\alpha|^2)]\{1 + \mathcal{O}(\tfrac{1}{|\alpha|^2})\}, \quad |\alpha|^2 >> 1, \tfrac{p}{2} \end{aligned} \tag{19}$$

The corresponding p-Gaussian (p-normal) distribution approximation to (18) is given in the appendix. The p-dependence in the p-Gaussian distribution for the pB coherent state arises only through the mean μ and the standard deviation σ. This is unlike the explicit p-dependence, and the explicit n_{even} versus n_{odd} differences, in the above p-Poisson distribution. From (18),$\frac{\mathcal{P}_p(2N+1)}{\mathcal{P}_p(2N)} = \frac{|\alpha|^2}{2N+p}$ and $\frac{\mathcal{P}_p(2N-1)}{\mathcal{P}_p(2N)} = \frac{2N}{|\alpha|^2}$. This means for the distribution of $|n >$'s in the coherent state $|\alpha >$ that versus the probability of an arbitrary $|n_{even} >$ number-state, the next "odd" number-state is less probable for $p > 1$ than for $p = 1$, but the probability for the previous "odd" is not p-dependent.

For two arbitrary coherent states,

$$< \alpha|\beta >= \sqrt{P_e(|\alpha|^2)P_e(|\beta|^2)} < \alpha_e|\beta_e > + \sqrt{P_o(|\alpha|^2)P_o(|\beta|^2)} < \alpha_o|\beta_o > \tag{20}$$

with

$$< \alpha_e|\beta_e >= \frac{e_e(\alpha^*\beta)}{\sqrt{e_e(|\alpha|^2)e_e(|\beta|^2)}};\ < \alpha_o|\beta_o >= \frac{e_o(\alpha^*\beta)}{\sqrt{e_o(|\alpha|^2)e_o(|\beta|^2)}};\ < \alpha_e|\beta_o >= 0 \tag{21}$$

So for large arguments: $|\alpha|, |\beta| >> 1, \frac{p-2}{2}$,

$$| < \alpha|\beta > | \to \exp(-\frac{1}{2}|\alpha-\beta|^2)\{1 + \mathcal{O}(\frac{1}{|\alpha|^2}, \frac{1}{|\beta|^2}, \frac{1}{|\alpha^*\beta|})\} \tag{22}$$

and for small arguments: $|\alpha|, |\beta| << 1$,

$$| < \alpha|\beta > | \to \{1 - \frac{|\alpha-\beta|^2}{2p} + \mathcal{O}(|\alpha|^4, |\beta|^4, |\alpha|^2|\beta|^2)\} \tag{23}$$

As a consequence of the completeness relation

$$I \equiv \frac{1}{\pi}\int d^2\alpha\ \mu_e(|\alpha|^2)\,|\alpha_e >< \alpha_e|\ + \frac{1}{\pi}\int d^2\alpha\ \mu_o(|\alpha|^2)\,|\alpha_o >< \alpha_o|\,, \tag{24}$$

where in terms both types of modified Bessel functions

$$\mu_e(|\alpha|^2) = |\alpha|^2 K_{\frac{p-2}{2}}(|\alpha|^2) I_{\frac{p-2}{2}}(|\alpha|^2);\ \mu_o(|\alpha|^2) = |\alpha|^2 K_{\frac{p}{2}}(|\alpha|^2) I_{\frac{p}{2}}(|\alpha|^2), \tag{25}$$

and of the $|\alpha >$ and $|\beta >$ non-orthogonality, the pB coherent states are linearly dependent. They are overcomplete. This relation[4] (24) follows by using

$$\begin{array}{l} \Gamma_e(n_e+1) \equiv (2N)_p! = \frac{2^{\frac{2-p}{2}}}{\Gamma(\frac{p}{2})}\int_0^\infty d(|\alpha|^2) K_{\frac{p-2}{2}}(|\alpha|^2)|\alpha|^{2(\frac{p}{2}+n_e)} \\ \Gamma_o(n_o+1) \equiv (2N+1)_p! = \frac{2^{\frac{2-p}{2}}}{\Gamma(\frac{p}{2})}\int_0^\infty d(|\alpha|^2) K_{\frac{p}{2}}(|\alpha|^2)|\alpha|^{2(\frac{p}{2}+n_o)} \end{array} \tag{26}$$

[4]By substitution of (17) into (24), eq(2.63) of 3rd paper in [6] is obtained.

This analytic integral representation for the two p-factorials generalizes Euler's formula for $\Gamma(x)$. Uses for generalizations of other functions of integers, e.g. $\zeta_p(k) \equiv \sum_{n_p=p}^{\infty}(n_p)^{-k}$, $k > 1$, remain to found.

In this $|\alpha_{e,o} >$ coherent state basis, there is a diagonal P_p-representation[5] for the density operator $\widehat{\rho}$ describing the state of the system

$$\widehat{\rho} \equiv \frac{1}{\pi}\int d^2\alpha \; \mu_e(|\alpha|^2)\, |\alpha_e ><\; \alpha_e|\, \Phi^{(e)}(\alpha) + \frac{1}{\pi}\int d^2\alpha \; \mu_o(|\alpha|^2)\, |\alpha_o ><\; \alpha_o|\, \Phi^{(o)}(\alpha) \tag{27}$$

For $\widehat{\rho} = \widehat{\rho}^\dagger$, $\Phi^{(e,o)}(\alpha)$ are real. The normalization condition from $\widehat{\rho} = \widehat{\rho}_e + \widehat{\rho}_o$ is

$$Tr\, \widehat{\rho}_{e,o} = I_{e,o} = \frac{1}{\pi}\int d^2\alpha \; \mu_{e,o}(|\alpha|^2)\, \Phi^{(e,o)}(\alpha) \tag{28}$$

For instance, if the system is in the coherent state $|\beta_e >$, then $\widehat{\rho}_{\beta_e} = |\beta_e ><\; \beta_e|$ for $\Phi^{(e)}(\alpha) = \frac{\pi}{\mu_e(|\beta|^2)}\delta^2(\alpha-\beta), \Phi^{(o)}(\alpha) = 0$ where $\delta^2(\alpha-\beta) = \delta(Re[\alpha-\beta])\delta(Im[\alpha-\beta])$.

To describe a field theoretic system in a maximum-entropic state, we proceed as in the treatment of ordinary bosons [2] for which such a field-state is often called a chaotic or thermal state: In terms of the paraboson number operator $\widehat{N} = \widehat{N}_B$, we maximize the entropy $S = -k\,Tr(\rho \ln \rho)$ to obtain

$$\widehat{\rho}_{\max S} = \frac{1}{1+ < \widehat{N} >}\left(\frac{< \widehat{N} >}{1+ < \widehat{N} >}\right)^{\widehat{N}} \tag{29}$$

where $< \widehat{N} >$ is the mean number of parabosons in the maximum-entropic state.

Defining $r \equiv 1 + \frac{1}{<\widehat{N}>}$, from the above P_p-representation, the corresponding non-negative maximum-entropic $\Phi^{(e,o)}(\alpha)$ functions are

$$\Phi^{(e)}_{\max S}(\alpha) = \frac{r^{\frac{p}{2}}}{< \widehat{N} >}\frac{K_{\frac{p-2}{2}}(r|\alpha|^2)}{K_{\frac{p-2}{2}}(|\alpha|^2)};\; \Phi^{(o)}_{\max S}(\alpha) = \frac{r^{\frac{p}{2}}}{< \widehat{N} >}\frac{K_{\frac{p}{2}}(r|\alpha|^2)}{K_{\frac{p}{2}}(|\alpha|^2)} \tag{30}$$

[5]This is the direct generalization of the bosonic P-representation, see [7]. We use $\Phi^{(e,o)}(\alpha)$ to denote the weight functions, so as to avoid confusion with other functions.

For ordinary bosons $\Phi^{(e)}_{\max S}(\alpha) = \Phi^{(o)}_{\max S}(\alpha) = \frac{1}{<\widehat{N}>} \exp(-|\alpha|^2/ < \widehat{N} >)$.

By using these results, in analogy with the scalar-field treatment for ordinary bosons, we can analyze Hanbury-Brown and Twiss intensity correlations [1,2] for a paraboson field which is in a single-mode maximum-entropic state: We consider zero-time-interval correlations. Using $\widehat{\rho}_{\max S} = \widehat{\rho}^{(e)}_{\max S} + \widehat{\rho}^{(o)}_{\max S}$, the first-order coherence function is

$$\begin{aligned} G^{(1)}(0) &\equiv Tr[\widehat{\rho}_{\max S}\ E^{(-)}(x_1) E^{(+)}(x_1)] \\ &= G^{(1)}_e(0) + G^{(1)}_o(0) \\ &= \overline{c} < \widehat{N} > \left[\tfrac{p+2<\widehat{N}>}{1+2<\widehat{N}>}\right] \end{aligned} \tag{31}$$

where $\overline{c}$ is a constant factor. The second-order, or intensity, correlation function is

$$\begin{aligned} G^{(2)}(0) &\equiv Tr[\widehat{\rho}_{\max S}\ E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_2) E^{(+)}(x_1)] \\ &= 2\ (\overline{c} < \widehat{N} >)^2 \left[\tfrac{p+2<\widehat{N}>}{1+2<\widehat{N}>}\right] \end{aligned} \tag{32}$$

If we write a proportionality $G^{(2)}(0) = \ \lambda_p\, [G^{(1)}(0)]^2$, then $\lambda_p = 2\ \left[\frac{1+2<\widehat{N}>}{p+2<\widehat{N}>}\right]$ which shows that as the mean number of parabosons decreases, there is a monotonic reduction to $\frac{2}{p}$ versus the usual bosonic "constant factor of two" intensity correlation effect.

Similarly, we obtain for the next higher-order correlation functions

$$G^{(3)}(0) = 3!(\overline{c} < \widehat{N} >)^3 \left[\frac{a\,p^2 + b\,p + c}{3(1 + 2 < \widehat{N} >)^3}\right] \tag{33}$$

$$G^{(4)}(0) = 4!(\overline{c} < \widehat{N} >)^4 \left[\frac{a\,p^2 + b\,p + c}{3(1 + 2 < \widehat{N} >)^3}\right] \tag{34}$$

where

$$\begin{aligned} a &= 1 + 2 < \widehat{N} > \\ b &= 2(1 + 4 < \widehat{N} > + 6 < \widehat{N} >^2) \\ c &= 8 < \widehat{N} > (1 + 3 < \widehat{N} > + 3 < \widehat{N} >^2) \end{aligned} \tag{35}$$

As in (31,32), the last factor in (33,34) depends on $< N >$; it approaches "1" for $< N >$ large, and a p-dependent value " $\frac{p^2+2p}{3}$ " as $< N > \rightarrow 0$.

Formulae for arbitrary n order ${G_{e,o}}^{(n)}(0)$ are listed in the appendix. There is no p-dependence in zero-time-interval, intensity correlation functions in the large $< \widehat{N} >$ limit. However, there is significant p-dependence as the mean number of parabosons decreases.

In summary, this paper shows that in intensity correlation measurements there will be clear and unambiguous signals that new quanta are, or aren't, parabosons. It will be a complete measurement, because the order p of their parastatistics will be determined!

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Note added in proof: Recursion relation for intensity correlation functions:

There is a simple recursion relation between the n_{even}-order and the lower-adjacent n_{odd}-order intensity correlation function

$$G^{(n_e)}(0) = \ n_e \ (\overline{c} < \widehat{N} >) \ G^{(n_e - 1)}(0).$$

This relation is independent of p and so it might be empirically very useful, for instance for measurement of the product of the constant factor and the mean number of parabosons, $(\overline{c} < \widehat{N} >)$. This relation follows from the hypergeometric function expressions in the appendix.

Appendix:

$G^{(n)}(0)$ **Formulae for Maximum-Entropic State:**

For $G^{(n)}(0) = G_e^{(n)}(0) + G_o^{(n)}(0)$ with $r = 1 + \frac{1}{<\widehat{N}>}$, there are integral representations

$$
\begin{aligned}
G_e^{(n_o)}(0) &= \frac{r^{\frac{p}{2}}}{<\widehat{N}>} \int_0^\infty dx\ x^{n_o+1}\, K_{\frac{p}{2}-1}(rx)\, I_{\frac{p}{2}}(x) \\
G_o^{(n_o)}(0) &= \frac{r^{\frac{p}{2}}}{<\widehat{N}>} \int_0^\infty dx\ x^{n_o+1}\, K_{\frac{p}{2}}(rx)\, I_{\frac{p}{2}-1}(x) \\
G_e^{(n_e)}(0) &= \frac{r^{\frac{p}{2}}}{<\widehat{N}>} \int_0^\infty dx\ x^{n_e+1}\, K_{\frac{p}{2}-1}(rx)\, I_{\frac{p}{2}-1}(x) \\
G_o^{(n_e)}(0) &= \frac{r^{\frac{p}{2}}}{<\widehat{N}>} \int_0^\infty dx\ x^{n_e+1}\, K_{\frac{p}{2}}(rx)\, I_{\frac{p}{2}}(x)
\end{aligned}
\tag{36}
$$

In this appendix, we suppress the overall $(\bar{c})^{n_i}$ factors. The $G_{e,o}^{(n_i)}(0)$ can be written in terms of the hypergeometric function or as an infinite series

$$
\begin{aligned}
G_e^{(n_o)}(0) &= \frac{2^{n_o}}{<\widehat{N}>r^{n_o+2}} \frac{\Gamma(\frac{p}{2}+\frac{n_o}{2}+\frac{1}{2})\Gamma(\frac{n_o}{2}+\frac{3}{2})}{\Gamma(\frac{p}{2}+1)}\ {}_2F_1(\tfrac{p}{2}+\tfrac{n_o}{2}+\tfrac{1}{2}, \tfrac{n_o}{2}+\tfrac{3}{2}; \tfrac{p}{2}+1; r^{-2}) \\
&= \frac{2^{n_o}}{<\widehat{N}>r^{n_o+2}} \sum_{m=0}^{\infty} \frac{\Gamma(\frac{p}{2}+N+1+m)\Gamma(N+2+m)}{\Gamma(\frac{p}{2}+1+m)\Gamma(m+1)}\ r^{-2m} \\
G_o^{(n_o)}(0) &= \frac{2^{n_o}}{<\widehat{N}>r^{n_o+1}} \frac{\Gamma(\frac{p}{2}+\frac{n_o}{2}+\frac{1}{2})\Gamma(\frac{n_o}{2}+\frac{1}{2})}{\Gamma(\frac{p}{2})}\ {}_2F_1(\tfrac{p}{2}+\tfrac{n_o}{2}+\tfrac{1}{2}, \tfrac{n_o}{2}+\tfrac{1}{2}; \tfrac{p}{2}; r^{-2}) \\
&= \frac{2^{n_o}}{<\widehat{N}>r^{n_o+1}} \sum_{m=0}^{\infty} \frac{\Gamma(\frac{p}{2}+N+1+m)\Gamma(N+1+m)}{\Gamma(\frac{p}{2}+m)\Gamma(m+1)}\ r^{-2m} \\
G_e^{(n_e)}(0) &= \frac{2^{n_e}}{<\widehat{N}>r^{n_e+1}} \frac{\Gamma(\frac{p}{2}+\frac{n_e}{2})\Gamma(\frac{n_e}{2}+1)}{\Gamma(\frac{p}{2})}\ {}_2F_1(\tfrac{p}{2}+\tfrac{n_e}{2}, \tfrac{n_e}{2}+1; \tfrac{p}{2}; r^{-2}) \\
&= \frac{2^{n_e}}{<\widehat{N}>r^{n_e+1}} \sum_{m=0}^{\infty} \frac{\Gamma(\frac{p}{2}+N+m)\Gamma(N+1+m)}{\Gamma(\frac{p}{2}+m)\Gamma(m+1)}\ r^{-2m} \\
G_o^{(n_e)}(0) &= \frac{2^{n_e}}{<\widehat{N}>r^{n_e+2}} \frac{\Gamma(\frac{p}{2}+\frac{n_e}{2}+1)\Gamma(\frac{n_e}{2}+1)}{\Gamma(\frac{p}{2}+1)}\ {}_2F_1(\tfrac{p}{2}+\tfrac{n_e}{2}+1, \tfrac{n_e}{2}+1; \tfrac{p}{2}+1; r^{-2}) \\
&= \frac{2^{n_e}}{<\widehat{N}>r^{n_e+2}} \sum_{m=0}^{\infty} \frac{\Gamma(\frac{p}{2}+N+1+m)\Gamma(N+1+m)}{\Gamma(\frac{p}{2}+1+m)\Gamma(m+1)}\ r^{-2m}
\end{aligned}
\tag{37}
$$

For $<\widehat{N}>\to \infty$, we assume the large $x = |\alpha|^2$ region of the integrand dominates, so $G_{e,o}^{(n)}(0) \to \frac{1}{2}n!\ <\widehat{N}>^n$ because of the absence of ν-dependence in $I_\nu(x)$ and $K_\nu(x)$ for large x.

Expansions in p follow from the infinite series expressions,

$$G_e^{(n_o)}(0) = \frac{2^N N! <\widehat{N}>^{2N+1}}{(1+2<\widehat{N}>)^{N+2}}\{p^{N+1}[1+2<\widehat{N}>]$$
$$+p^N(N+1)[N+2N<\widehat{N}> +2(N+1)<\widehat{N}>^2]+\cdots\}$$
$$G_o^{(n_o)}(0) = \frac{2^N N! <\widehat{N}>^{2N+1}}{(1+2<\widehat{N}>)^{N+2}}\{p^N(N+1)[2<\widehat{N}> +2<\widehat{N}>^2]+\cdots\}$$
$$G^{(n_o)}(0) = \frac{2^N N! <\widehat{N}>^{2N+1}}{(1+2<\widehat{N}>)^{N+2}}\{p^{N+1}[1+2<\widehat{N}>]$$
$$+p^N(N+1)[N+2(N+1)<\widehat{N}> +2(N+2)<\widehat{N}>^2]+\cdots\} \tag{38}$$

and

$$G_e^{(n_e)}(0) = \frac{2^N N! <\widehat{N}>^{2N+1}}{(1+2<\widehat{N}>)^{N+2}}\{p^N[1+2<\widehat{N}>]$$
$$+p^{N-1}(N)[N+1+2(N+1)<\widehat{N}> +2(N+1)<\widehat{N}>^2]+\cdots\}$$
$$G_o^{(n_e)}(0) = \frac{2^N N! <\widehat{N}>^{2N}(1+<\widehat{N}>)}{(1+2<\widehat{N}>)^{N+2}}\{p^N[1+2<\widehat{N}>]$$
$$+p^{N-1}(N)[N-1+2(N-1)<\widehat{N}> +2(N+1)<\widehat{N}>^2]+\cdots\}$$
$$G^{(n_e)}(0) = \frac{2^N N! <\widehat{N}>^{2N}}{(1+2<\widehat{N}>)^{N+1}}\{p^N[1+2<\widehat{N}>]$$
$$+p^{N-1}(N)[N-1+2N<\widehat{N}> +2(N+1)<\widehat{N}>^2]+\cdots\} \tag{39}$$

p-Gaussian Distribution:

Per the central limit theorem, the p-Gaussian distribution (p-normal) approximation to the p-Poisson distribution follows from (18):

In the p-Poisson distribution function $\mathcal{P}_p(n_i, x)$ for large $x = |\alpha|^2$, we change variables to $y = \frac{n-\mu}{\sigma}$, so as to measure the deviation of n versus the mean μ in units of the standard deviation σ.

$$\mu \equiv <\alpha|\widehat{N}|\alpha>$$
$$= |\alpha|^2 + \frac{1}{2}(1-p) - \frac{D}{2}(1-p) \tag{40}$$

$$\sigma^2 \equiv < \alpha|(\widehat{N}-\mu)^2|\alpha > \\ = |\alpha|^2 + \tfrac{1}{2}(1-p)^2 + D(1-p)|\alpha|^2 - \tfrac{1}{4}D^2(1-p)^2 \tag{41}$$

Both depend on the positive difference between the $n_{e,o}$ coherent-state-mode probabilities of (14)

$$D = D(|\alpha|^2) \equiv P_e(|\alpha|^2) - P_o(|\alpha|^2) = \frac{[I_{\frac{p-2}{2}}(|\alpha|^2) - I_{\frac{p}{2}}(|\alpha|^2)]}{[I_{\frac{p-2}{2}}(|\alpha|^2) + I_{\frac{p}{2}}(|\alpha|^2)]} > 0. \tag{42}$$

For $|\alpha|^2 >> 1, \frac{p}{2}$,

$$\mu \simeq |\alpha|^2 + \tfrac{1}{2}(1-p) \\ \sigma^2 \simeq |\alpha|^2 + \tfrac{1}{2}(1-p)^2 \tag{43}$$

since $D \ \to 0$. For $|\alpha|^2 \to 0$, $D \to 1 - \frac{2|\alpha|^2}{p} + \mathcal{O}(|\alpha|^4)$.

Expanding and using the Stirling approximation, we obtain as in the $p = 1$ case [8]

$$\mathcal{P}_p(n_e, y) = \tfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}y^2} [\ 1 - \tfrac{1}{\sigma}(\tfrac{y}{2} - \tfrac{y^3}{6}) \\ -\tfrac{1}{\sigma^2}(\tfrac{1}{12} + \tfrac{p}{4} - \tfrac{p^2}{4} - y^2\{\tfrac{1}{8} + \tfrac{p}{2} - \tfrac{p^2}{4}\} + \tfrac{y^4}{6} - \tfrac{y^6}{72}) \\ + \mathcal{O}(\tfrac{1}{\sigma^3})\ ];\ y \equiv \tfrac{n-\mu}{\sigma} \tag{44}$$

For $\mathcal{P}_p(n_o, y)$ replace " $\frac{1}{12} + \frac{p}{4}$ " in the σ^{-2} coefficient by " $-\frac{5}{12} + \frac{3p}{4}$ ". In the bosonic case, additional terms through σ^{-4} coefficients are given in [8].

As for $p = 1$, using the first term of this series, we define the p-Gaussian distribution (p-normal)

$$\mathcal{G}_p(n, \mu, \sigma) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{n-\mu}{\sigma})^2] \tag{45}$$

where μ, σ for the pB coherent state are given in (40,41). Unless $\frac{p^2}{\sigma^2}$ and/or $\frac{(yp)^2}{\sigma^2}$ is large, for $|\alpha|^2$ large the p-Gaussian distribution will be a satisfactory approximation to the p-Poisson when $\frac{y^3}{\sigma}$ is small.

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